

Zero-field spin splitting in a two-dimensional electron gas with the spin-orbit interaction revisited

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We consider a two-dimensional electron gas (2DEG) with the Rashba spin-orbit interaction (SOI) in presence of a perpendicular magnetic field. We derive analytical expressions of the density of states (DOS) of a 2DEG with the Rashba SOI in presence of magnetic field by using the Green's function technique. The DOS allows us to obtain the analytical expressions of the magnetoconductivities for spin-up and spin-down electrons. The conductivities for spin-up and spin-down electrons oscillate with different frequencies and gives rise to the beating patterns in the amplitude of the Shubnikov de Hass (SdH) oscillations. We find a simple equation which determines the zero-field spin splitting energy if the magnetic field corresponding to any beat node is known from the experiment. Our analytical results reproduce well the experimentally observed non-periodic beating patterns, number of oscillations between two successive nodes and the measured zero-field spin splitting energy.

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I. INTRODUCTION

There has been tremendous research interest on the spin-orbit interaction (SOI) in low-dimensional condensed matter systems due to the possible applications in spin based electronic devices¹⁻³. The SOI is responsible for many noble effects like spin-FET^{4,5}, metal-insulator transition in a two-dimensional hole gas⁶, spin-resolved ballistic transport⁷, spin-galvanic effect⁸ and spin Hall effect^{9,10}. It is the prime interest to measure and control the SOI strength experimentally because it influences the spin degree of freedom.

In absence of an external magnetic field, the two-fold spin degeneracy is lifted at finite momentum of the electron due to the SOI. In semiconductor heterostructures, there are mainly two mechanisms responsible for giving rise to the zero-field spin splitting: the Dresselhaus interaction¹¹ which varies as k^3 and the Rashba interaction¹² which varies linear with k . The former is due to the inversion asymmetry of the crystals and the later is due to the asymmetric quantum wells. The Dresselhaus interaction dominates in the wide-gap semiconductors with small thickness whereas the Rashba interaction dominates in narrow-gap semiconductors due to their different momentum dependence^{13,14}. The electric field generated in the asymmetric quantum wells produces the Rashba spin-orbit interaction. The SOI can also be tuned by a strong external electric field perpendicular to the planar motion of the electrons.

A direct manifestation of the spin-split levels due to the SOI is a beating pattern in the SdH oscillations due to the two closely spaced different frequency of the spin-up and spin-down electrons. In Ref.¹⁴, it was shown that the Rashba interaction produces regular beating patterns in the SdH oscillations whereas the Dresselhaus interaction produces anomalous beating patterns. The SOI strength is measured by analyzing the beating patterns in the SdH oscillations.

The experimental evidence, by using the magneto-

transport and cyclotron resonance measurement, of zero-field spin splitting is found in a modulation doped GaAs/AlGaAs heterojunction^{15,16}. It was first explained by Bychkov and Rashba based on the spin-orbit interaction produced in the asymmetric quantum wells¹⁷. The zero-field spin splitting is related to the Fermi wave vector k_F and the SOI strength α as $\Delta_s = 2k_F\alpha$. The Rashba SOI is considered to be the appropriate term for observing the zero-field spin splitting in low-dimensional quantum systems, particularly in narrow gap semiconductors. In Ref.¹⁸, the beating patterns in the SdH oscillations have been found in GaSb/InAs quantum wells and confirmed that the lifting of the spin degeneracy is due to the Rashba SOI. Later, Das et al.¹⁹ investigated the SdH oscillations in a series of three different modulation-doped heterostructures with high electron densities and confirmed that the lifting of the spin degeneracy is due to the Rashba SOI. There are many advanced technique to control the SOI strength in 2DEG of different materials by varying the gate voltage²⁰⁻²².

In Ref.²³, the zero-field spin splitting energy and hence the Rashba SOI strength were determined by extrapolating data and by using the model calculations for the SdH oscillations. Later, the zero-field spin splitting energy was studied theoretically based on the difference between the Landau energy levels²⁴ and the self-consistent Born approximation²⁵. The estimated SOI strength was in good agreement with the extrapolated results obtained in¹⁹.

In this work, we provide a very simple and elegant way to determine the zero-field spin splitting energy. The analytical expressions of the SdH oscillations for the spin-up and spin-down electrons are also obtained. The total magnetoconductivity shows beating patterns in the SdH oscillations due to two closely spaced different frequencies of the SdH oscillations for spin-up and spin-down electrons. By analyzing the beating patterns, we find a very simple equation to determine the zero-field spin splitting energy from the location of any beat node. We

also explain analytically the non-periodic behavior of the beating patterns and the number of oscillations between two successive nodes.

This paper is organized as follows. In section II, we summarize the exact energy eigenvalues and the corresponding eigenfunctions of a 2DEG with the Rashba SOI in presence of a perpendicular constant magnetic field. We calculate the DOS of a 2DEG with the SOI in presence of a magnetic field. Numerical and analytical results of the SdH oscillations are presented in section III. In section V, we use available experimental data to calculate the zero-field spin splitting energy, number of oscillations between two nodes from our analytical results. We present a summary of our work in Sec. VI.

II. ENERGY SPECTRUM AND DENSITY OF STATES IN PRESENCE OF RASHBA SOI

The Hamiltonian of an electron ($-e$) with the Rashba SOI in presence of a perpendicular magnetic field $\mathbf{B} = B\hat{z}$ is given by

$$H = \frac{(\mathbf{p} + e\mathbf{A})^2}{2m^*} \mathcal{K} + \frac{\alpha}{\hbar} [\boldsymbol{\sigma} \times (\mathbf{p} + e\mathbf{A})]_z + \frac{1}{2} g \mu_B B \sigma_z, \quad (1)$$

where \mathbf{p} is the 2D momentum operator, m^* is the effective mass of the electron, g is the Lande-g factor, μ_B is the Bohr magneton, \mathcal{K} is the unit matrix, $\boldsymbol{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$ are the Pauli spin matrices, and α is the strength of the Rashba interaction.

Here, we shall just mention the exact solutions of the Hamiltonian H . Using the Landau wave functions without the Rashba SOI as the basis, one can obtain the energy spectrum and the corresponding eigenfunctions²⁴. The resulting eigenstates are labeled by a new quantum number s , instead of the Landau level quantum number n in absence of the SOI. For $s = 0$ there is only one level, the same as the lowest Landau level without SOI, with energy $E_0^+ = E_0 = (\hbar\omega - g\mu_B B)/2$ and the corresponding wave function is

$$\Psi_0^+(k_y) = \frac{e^{ik_y y}}{\sqrt{L_y}} \phi_0(x + x_0) \begin{pmatrix} 0 \\ 1 \end{pmatrix}. \quad (2)$$

Here, $\omega = eB/m^*$, $x_0 = k_y l_0^2$ with $l_0 = \sqrt{\hbar/(eB)}$ is the magnetic length scale and $\phi_0(x) = e^{-x^2/(2l_0^2)}/\sqrt{\sqrt{\pi}l_0}$. For $s = 1, 2, 3, \dots$ there are two branches of the energy levels, denoted by $+$ corresponding to the "spin-up" electrons and $-$ corresponding to the "spin-down" electrons with energies

$$E_s^\pm = s\hbar\omega \pm \sqrt{E_0^2 + sE_\alpha \hbar\omega}, \quad (3)$$

where $E_\alpha = 2m^*\alpha^2/\hbar^2$ is the Rashba energy determined by the SOI strength α . The corresponding wave function for $+$ branch is

$$\Psi_s^+(k_y) = \frac{e^{ik_y y}}{\sqrt{L_y A_s}} \begin{pmatrix} D_s \phi_{s-1}(x + x_0) \\ \phi_s(x + x_0) \end{pmatrix}, \quad (4)$$

and the $-$ branch is

$$\Psi_s^-(k_y) = \frac{e^{ik_y y}}{\sqrt{L_y A_s}} \begin{pmatrix} \phi_{s-1}(x + x_0) \\ -D_s \phi_s(x + x_0) \end{pmatrix}, \quad (5)$$

where $A_s = 1 + D_s^2$, $D_s = \sqrt{sE_\alpha \hbar\omega}/[E_0 + \sqrt{E_0^2 + sE_\alpha \hbar\omega}]$. Also, $\phi_s(x) = e^{-x^2/(2l_0^2)} H_s(x/l_0)/\sqrt{\sqrt{\pi} 2^s s! l_0}$ is the harmonic oscillator wavefunctions and $H_s(x)$ is the Hermite polynomial.

We calculate the density of states (DOS) $D(E)$ of a 2DEG with the Rashba SOI in presence of a weak magnetic field. This is calculated by taking imaginary part of the electron's self-energy $\Sigma^-(E)$ using the expression $D(E) = \text{Im} \{ \Sigma^-(E)/(\pi^2 l_0^2 \Gamma_0^2) \}$. Here, Γ_0 is the impurity induced Landau level broadening. The DOS for spin-up and spin-down electrons are calculated and these are given as

$$D^\pm(E) = \frac{m^*}{2\pi\hbar^2} \left[1 + 2 \exp \left\{ -2 \left(\frac{\pi\Gamma_0}{\hbar\omega} \right)^2 \right\} \times \cos \left\{ \frac{2\pi}{\hbar\omega} \left(E + \frac{E_\alpha}{2} \mp \sqrt{E_0^2 + E_\alpha E} \right) \right\} \right]. \quad (6)$$

The analytical expressions of the DOS given in Eq. (6) will be used to calculate the magnetoconductivity in the next section.

In absence of the Rashba and the Zeeman terms, the above mentioned DOS reduces to the well-known result of the DOS of a 2DEG without SOI in presence of the magnetic field^{26,27}:

$$D(E) = \frac{m^*}{\pi\hbar^2} \left[1 - 2 \exp \left\{ -2 \left(\frac{\pi\Gamma_0}{\hbar\omega} \right)^2 \right\} \cos \left(\frac{2\pi E}{\hbar\omega} \right) \right]. \quad (7)$$

III. CALCULATION OF MAGNETOCONDUCTIVITY

In general, there are two scattering mechanisms, diffusive and collisional scattering, contribute to the transport properties. The diffusive scattering is due to finite drift velocity gained by the electrons. In our case there is no finite group velocity along y -direction due to the k_y degeneracy in the energy spectrum. Therefore the diffusive contribution to the total conductivity is zero. The collisional conductivity arises because of the migration of the cyclotron orbit due to scattering from the charge impurities present in the system. In our problem, the diagonal conductivity $\sigma_{xx} = \sigma_{xx}^{\text{col}}$ because $\sigma_{xx}^{\text{dif}} = \sigma_{yy}^{\text{dif}} = 0$. The magnetoresistivity is $\rho_{yy} = \sigma_{xx}/S$, where $S = \sigma_{xx}\sigma_{yy} - \sigma_{xy}\sigma_{yx} = \sigma_{xy}^2$ with $\sigma_{xy} \simeq n_e e/B$.

At low temperature, we assume that electrons are scattered elastically by charged impurities distributed uniformly. The standard expression for collisional conductivity²⁸ is given by

$$\sigma_{\mu\mu}^{\text{col}} = \frac{\beta e^2}{S_0} \sum_{\xi, \xi'} f_\xi (1 - f_\xi) W_{\xi, \xi'} (\alpha_\mu^\xi - \alpha_\mu^{\xi'})^2, \quad (8)$$

where $W_{\xi,\xi'}$ is the transition between one-electron states $|\xi\rangle$ and $|\xi'\rangle$. Also, $\alpha_\mu^\xi = \langle \xi | r_\mu | \xi \rangle$ is the expectation value of the μ component of the position operator when the electron is in the state $|\xi\rangle$. The scattering rate $W_{\xi,\xi'}$ is given by

$$W_{\xi,\xi'} = \sum_{\mathbf{q}_0} |U(\mathbf{q}_0)|^2 \langle \xi | e^{i\mathbf{q}_0 \cdot (\mathbf{r}-\mathbf{R})} | \xi' \rangle^2 \delta(E_\xi - E_{\xi'}), \quad (9)$$

where $\mathbf{q}_0 = q_{0x}\hat{x} + q_{0y}\hat{y}$ is the 2D wave-vector and $U(\mathbf{q}_0) = 2\pi e^2 / (\epsilon \sqrt{q_{0x}^2 + q_{0y}^2 + k_s^2})$ is the Fourier transform of the screened impurity potential $U(\mathbf{r}) = (e^2/4\pi\epsilon)(e^{-k_s r}/r)$, where k_s is the inverse screening length and ϵ is the dielectric constant of the material. In the limit of small $|\mathbf{q}_0| \ll k_s$, $U(\mathbf{q}_0) \simeq 2\pi e^2 / (\epsilon k_s) = U_0$. Here, \mathbf{r} and \mathbf{R} are the position vector of electron and impurity, respectively. Finally the conductivity for spin-up and spin-down electrons become²⁴

$$\sigma_{xx}^\pm = \frac{e^2}{h} \frac{\beta N_I U_0^2}{2\pi \Gamma_0 l_0^2} \sum_s I_s^\pm f_s^\pm (1 - f_s^\pm), \quad (10)$$

where N_I is the 2D impurity number density and f_s^\pm is the Fermi-Dirac distribution function. The exact expressions of I_s^\pm are given as $I_s^\pm = [(2s \mp 1)D_s^4 - 2sD_s^2 + (2s \pm 1)]/A_s^2$.

We would like to derive analytical expressions of the conductivities for spin-up and spin-down electrons by using the DOS given in Eq. (6). The summation over quantum number s in Eq. (10) can be replaced as $\sum_s \rightarrow 2\pi l_0^2 \int_0^\infty D(E)dE$. After a lengthy calculation, we obtain the analytical expressions of the conductivities for spin-up and spin-down electrons, which are given by

$$\begin{aligned} \frac{\sigma_{xx}^\pm}{\sigma_0} &= \frac{\tilde{E}_F}{8(\omega\tau)^2} \left[1 + 2 \exp \left\{ -2 \left(\frac{\pi\Gamma_0}{\hbar\omega} \right)^2 \right\} \right] \\ &\times A \left(\frac{T}{T_c} \right) \cos \left(\frac{2\pi f^\pm}{B} \right), \end{aligned} \quad (11)$$

where $\sigma_0 = n_e e^2 \tau / m^*$ is the Drude conductivity, $\tilde{E}_F = \left[1 + \frac{E_\alpha}{2E_F} \mp \frac{3}{2} \sqrt{\frac{E_\alpha}{E_F}} \right]$, $A(T/T_c) = (T/T_c) / \sinh(T/T_c)$ with $T_c = \hbar\omega / 2\pi^2 k_B$. The conductivities for spin-up and spin-down electrons are oscillating with different frequencies f^\pm (in Tesla) as given below:

$$f^\pm = \frac{m^*}{\hbar e} \left[E_F + \frac{E_\alpha}{2} \mp \sqrt{E_0^2 + E_\alpha E_F} \right]. \quad (12)$$

The total conductivity is given by

$$\begin{aligned} \frac{\sigma_{xx}}{\sigma_0} &\simeq \frac{1}{4(\omega\tau)^2} \left[1 + 2 \exp \left\{ -2 \left(\frac{\pi\Gamma_0}{\hbar\omega} \right)^2 \right\} \right] A \left(\frac{T}{T_c} \right) \\ &\times \cos(2\pi \frac{f_a}{B}) \cos(2\pi \frac{f_d}{B}), \end{aligned} \quad (13)$$

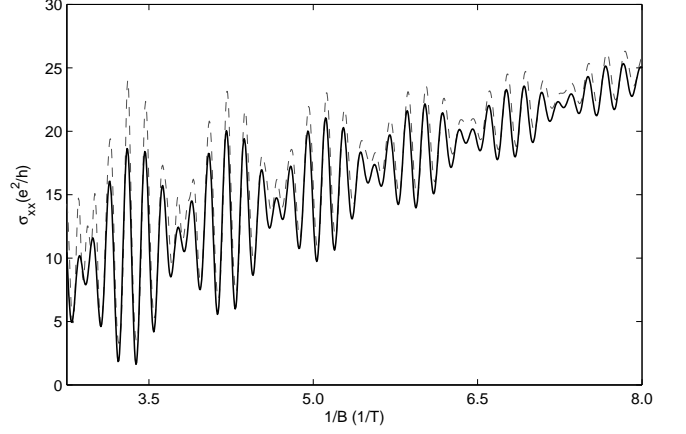


FIG. 1: Plots of the exact (dashed) and analytical (solid) expression of the total conductivities vs inverse magnetic field B .

where $f_a = (f^+ + f^-)/2$ and $f_d = (f^+ - f^-)/2$. It clearly shows that the total conductivity produces beating patterns in the amplitude of the SdH oscillations. In Fig. 1, we compare the analytical result of the total conductivity with the exact numerical results obtained from Eq. (10). The beating pattern obtained from the analytical expression is in excellent agreement with the numerical results, particularly the locations of the nodes. For Fig. 1, the following parameters are used: $\alpha = 10^{-11}$ eV-m, $\Gamma_0 = 0.02$ meV, electron density $n_e = 3 \times 10^{15}$ /m², electron effective mass $m^* = 0.05m_0$ with m_0 is the free electron mass, and temperature $T = 1$ K.

The number of oscillations between two successive nodes is

$$N_{osc} = f_a \Delta \left(\frac{1}{B} \right) = \frac{m^*}{\hbar e} \left(E_F + \frac{E_\alpha}{2} \right) \left(\frac{1}{B_{j+1}} - \frac{1}{B_j} \right), \quad (14)$$

where B_j is the magnetic field corresponding to the j -th node.

The last cosine term, $\cos(2\pi f_d/B)$ is not periodic in $1/B$ because the frequency difference, $f_d = \frac{m^*}{\hbar e} \sqrt{E_0^2 + E_\alpha E_F}$, itself depends on the magnetic field B . The non-periodic behavior of the beating patterns observed in the experiments is due to the magnetic field dependence of the term f_d .

At the node positions $B = B_j$, we have the following condition: $\cos(2\pi f_d/B)|_{B=B_j} = 0$, which gives us

$$\sqrt{4E_0^2 + \Delta_s^2} = \hbar\omega_j \left(j + \frac{1}{2} \right), \quad (15)$$

where $\Delta_s = 2k_F\alpha$ is the zero-field spin splitting energy, $j = 1, 2, 3, \dots$ are the j -th beat node and $\omega_j = eB_j/m^*$. The zero-field spin splitting energy can be easily evaluated by knowing the magnetic field corresponding to any

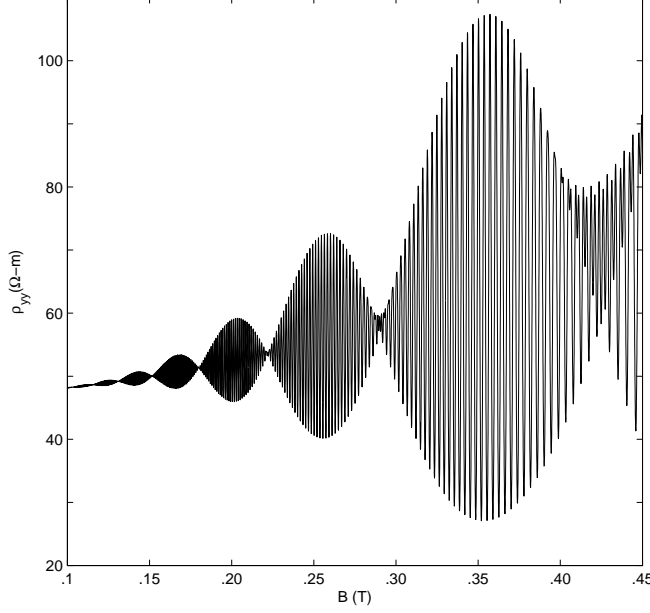


FIG. 2: Plots of the total resistivity vs magnetic field B .

beat node. The above equation can be re-written as

$$B_j = \frac{2m^*}{e\hbar} \frac{\Delta_s}{\sqrt{(2j+1)^2 - (g-2)^2}}. \quad (16)$$

The above equation is exactly the same as given in Ref.²⁵, which has been obtained by using the self-consistency Born approximation.

A. Comparison with Experiment

In this section we would like to test our analytical expressions by calculating the positions of the nodes, the zero-field spin splitting energy and the number of oscillations between two nodes and then comparing with the experimental observations. To reproduce the experimental observations of the beating patterns in the SdH oscillations¹⁹, we plot the resistivity as a function of the magnetic field in Fig. 2. For Fig. 2, we have taken the following parameters as used in the experiment¹⁹ for sample A: $\alpha = 3.76 \times 10^{-12}$ eV-m, $\Gamma_0 = 1.5$ meV, electron density $n_e = 1.75 \times 10^{16}$ /m², electron effective mass $m^* = 0.046m_0$ with m_0 is the free electron mass and temperature $T = 1.5$ K. It is interesting to note that the locations of the beat nodes and the number of oscillations between two successive nodes match very well with the experimental observations¹⁹.

We would like to determine the zero-field spin splitting energy and hence the Rashba SOI strength by using the positions of the beat nodes for three different samples

A, B, and C used in Ref.¹⁹. In Ref.¹⁹, the following parameters are used: $m^* = 0.046m_0$ and $n_e = 1.75 \times 10^{16}$ m⁻², 1.65×10^{16} m⁻², 1.46×10^{16} m⁻² for the A, B, and C samples, respectively. Using Eq. (15), we determine the zero-field spin splitting energy and the SOI strength for the first six nodes as given in the Table I, The av-

node	Beat points (B) in Tesla	Δ_s^A meV	α (10^{-12}) eV-m
1	0.873	2.44	3.69
2	0.460	2.63	3.98
3	0.291	2.43	3.68
4	0.227	2.49	3.76
5	0.183	2.47	3.74
6	0.153	2.45	3.71

TABLE I: Sample A: Zero-field spin splitting energy and the Rashba SOI strength α for different positions of the beat nodes.

erage values of the zero-field spin splitting energy at the Fermi level is $\Delta_s^A = 2.49$ meV and the Rashba SOI is 3.76×10^{-12} eV-m. Our results are nearly same as obtained in Ref.¹⁹. Similarly for the sample B, we obtain the following results given in the Table II:

node	Beat points (B) in Tesla	Δ_s^B meV	α (10^{-12}) eV-m
2	0.294	2.69	4.19
3	0.200	2.69	4.19
4	0.152	2.67	4.15
5	0.128	2.77	4.31
6	0.103	2.67	4.15

TABLE II: Sample B: Zero-field spin splitting energy and the Rashba SOI strength α for different positions of the beat nodes.

The average value of the zero-field spin splitting energy is $\Delta_s^B = 2.69$ meV and the Rashba SOI strength is $\alpha = 4.19 \times 10^{-12}$ eV-m. Similarly, for sample C, we obtain the average value of the zero-field spin splitting energy is $\Delta_s^C = 1.76$ meV and the Rashba SOI strength is $\alpha = 2.91$ eV-m. These results are in excellent agreement with the result obtained in Ref.¹⁹.

Now we consider another experiment where the SOI strength has been measured¹⁸. By knowing the positions of the two successive node points, we can also calculate the SOI strength. Using the parameters used in Ref.¹⁸, we obtain the Rashba SOI strength $\alpha = 0.9 \times 10^{-9}$ eV-cm which is exactly the same as obtained in Ref.¹⁸.

The number of oscillations between two successive nodes $j = 1$ and $j = 2$ counted from Ref.¹⁹ is 36 [see figure of Ref.¹⁹]. Using the parameters used in the experiment¹⁹, we obtain $N_{osc} = 37$ which exactly matches with the experimental observations. We consider another experiment²⁰ where the SOI strength was

varied by varying the gate voltage. For the gate voltages $V_g = 0.3$ and 1.5 Volt, the calculated number of oscillations between two nodes are $N_{osc} = 27$ and 30 , respectively. These numbers are the same with the direct observations.

IV. CONCLUSION

In this work, the beating patterns in the SdH oscillations in a 2DEG with the Rashba SOI is revisited. We have derived the DOS for spin-up and spin-down electrons in a 2DEG with the Rashba SOI in presence of a magnetic field analytically. The analytical expressions of the DOS will be very useful to calculate other properties, like magnetization, susceptibility etc, of a 2DEG with the SOI analytically. We have provided analytical expressions of the magnetoconductivities for spin-up and spin-down electrons, which oscillate with two closely different frequencies. The frequencies of the conductiv-

ity oscillations depend on the electron density, the SOI strength and also the external magnetic field. We have used the most conventional approach to get the simple equation which determines the zero-field spin splitting energy by knowing the magnetic field corresponds to any beat node. The number of oscillations in any beat can be easily found from our expression by knowing the two beat points. The calculated number of oscillations in a beat exactly matches with the experimental findings. The non-periodic beating pattern is due to the magnetic field dependence of the frequency difference between spin-up and spin-down electrons.

V. ACKNOWLEDGEMENT

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